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A model of political voting behaviours across different countries

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HIGHLIGHTS

- From the data, two types of general voting behaviours are found.
- A three-state Markov model provides a good fit for the time dependent data.
- Phenomenological behaviours are tested on a stochastic model reproducing the data.
- Both leader and social network influences are required to understand voter behaviour.

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ABSTRACT

This paper analyses, models mathematically, and compares national voting behaviours across seven democratic countries that have a long term election history, focusing on re-election rates, leaders' reputation with voters and the importance of friends' and family influence. Based on the data, we build a Markov model to test and explore national voting behaviour, showing voters are only influenced by the most recent past election. The seven countries can be divided into those in which there is a high probability that leaders will be re-elected and those in which incumbents have relatively less success.

A simple stochastic phenomenological dynamical model of electoral districts in which voters may be influenced by social neighbours, political parties and political leaders is then created to explore differences in voter behaviours in the countries. This model supports the thesis that an unsuccessful leader has a greater negative influence on individual voters than a successful leader, while also highlighting that increasing the influence on voters of social neighbours leads to a decrease in the average re-election rate of leaders, but raises the average amount of time the dominant party is in charge.

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1. Introduction

Understanding national voting behaviour and analysing various influences on voters has a long history of research in terms of developing fair election systems [1–4], but also due to it being a complex system [5–7]. In order to explore these various influences, several models (both sociological and mathematical) have been proposed looking at basic forms and constitutions of Government [8–12], voter systems [13–17] and election behaviours [18–24].

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Research on voting behaviour [25–30] has shown that the reputation of a political party's leader has an influence on voters. Studies frequently show that a leader's unsuccessful reputation reduces the chances of re-election more than a successful one enhances them [25,27,29,31]. Voters' perceptions of leadership also shape their evaluation of party policy [26–30].

Differences between voting behaviour in various countries have also been observed due to a variety of cultural and constitutional differences. In the past, voters in the United Kingdom and Australia have tended to be more influenced by factors including class, age, gender, religion, and ethnicity [32–34], whereas western European voters tend to elect parties that present clear political alternatives [16]. On the other hand, it has been shown in the past that American voters were strongly influenced by their family's party preferences [35]. It has also been shown that the type of election process used has a stronger influence on voting behaviour than historical or cultural background [24]. Hence, there is a complex interaction of factors and influences determining the outcome of individual elections.

In order to untangle this intricate mix of reasons for voter behaviour, several simple conceptual voting models have been proposed. One of the first was proposed by Campbell et al. [36], who used the layout of the funnel of causality model to describe voting behaviour in a two-party system at a point in time, such that current understanding of voting influences can be understood. In Campbell et al.'s model, voters inherit their political party preference from their parents, which influences the individual's future affiliation's to a party, the issues of domestic and foreign policy and also compares how the two parties deal with governmental affairs. This simple model was able to predict 87% of voting decisions, suggesting that American voters are significantly more influenced by their families than by party policy. However, Campbell et al.'s conclusions have been criticised on the grounds that they used unrepresentative election results from the 1950s and that voters are modelled as uninformed and lacking interest in politics, with the result that every voter's party preference was represented as being entirely dependent on loyalty towards a party to the exclusion of other individual influences, such as class, age and gender [37]. Furthermore, national influences such as the perception of a leader or media are not included but are known to have major impacts on voter behaviours; see Ref. [25]. Despite all these issues, Campbell et al.'s model is widely used due to it explaining a large part of American voter behaviour and supporting the hypothesis that American voters are "mostly" influenced by their immediate social network.

Galam [19–21,38] proposed several simple mathematical voter models in order to investigate the effect of various individual influences on group decision and voting. Instead of analysing party preferences, his hierarchical model analyses the election process where voters choose one of the two proposed policies. Random voters are then selected to represent the policy in the above hierarchical level, where they gather together to form various groups. Each voter selected keeps their policy preference, but again, one of the elected in each group is randomly chosen and put forward into the following level. This process continues until only one member is elected to represent their policy [19,38]. Galam proposed using the statistical physics Ising model to describe how voters make policy choices [20]. In the simplest form of this model, the voters can choose between one of two policies and a voter's preference is influenced by an initial choice and the votes of their closest neighbours, e.g. family/friends. It is found that depending on the strength of the influence of a voter's immediate social network one either observes the group of voters selecting one policy with a 50% preference, or (for strong influence) regions where voters all vote for one policy, i.e. segregation.

While these voting models have allowed researchers to investigate some of the fundamental underlying mechanisms for voter behaviour, the models generally only capture local individual influences on voter behaviour and ignore the national influence of media/leaders. Furthermore, we wish to develop models that explore conceptual mechanisms for voter behaviour while also being able to reproduce gross averages of the election data.

The aim of this paper is to first look at election rates of seven different countries with a long democratic election history. Although it is known that the election type does have an impact on voting behaviour [24], these have only been imposed in the past few decades. Therefore for us to analyse a larger set of election results, this research ignores the precise type of election process. We will concentrate on election averages rather than individual elections throughout this paper hence the effect of precise election policies/manifestos will be significantly reduced. We carry out a detailed data analysis by fitting a simple three-state Markov model to the election data describing the transition from first-time elected, re-election once and re-elected multiple times of individual leaders and parties, which suggests that a fundamental mechanism for voter behaviour is based on how many times a leader or party has been elected and that a suitable time-scale is per election. Furthermore, we are able to carry out a detailed analysis and comparison of voting behaviour in the seven different countries. Since we concentrate on election averages and have largely ignored policies/constitutions, one possible explanation for the similarities and differences seen in the voting behaviour is due to the influence of social network and a leader's reputation on individual voters. In order to investigate this hypothesis, we construct a voting model that incorporates the fundamental voter mechanism analysis with the influence of family/friends/etc. and a leader's reputation on voters using a 2-D Galam's model as a basis. The voting model carries out time steps every election and is inline with the time-scale found from the Markov model analysis. We then compare our voting model to the election averages to investigate different national voting behaviours.

Neither the Markov model or voting model takes the choice of constitution into account, even though it is known that it does impact the voting turn out and indirectly the national voting behaviour. While our models will be very simple, we find that we can draw several important conclusions about various national voting behaviour and some basic voting mechanisms. Crucially, we note that we are not attempting to predict a single election but rather explain some common mechanisms for voter behaviour that can explain the similarities and differences seen across different countries.

The outline of the paper is as follows. With the aid of a Markov model, Section 2 analyses the data on the seven national election histories. In Section 3 a voting model is constructed and simulated to investigate the combined effects of a leader's

Table 1
A list of the percentages in terms of elections for the five original Commonwealth countries.

	Australia	Canada	Rep. of Ireland	New Zealand	UK
Dominant party in charge (%)	61.5	57	58	62.5	60
Chance of a leader being re-elected (%)	51	63	53	66	47

Table 2
A list of the percentages gathered by the election results from France and the USA.

	France	USA
Dominant party in charge (%)	51.8	65.9
Chance of a leader being re-elected (%)	21.4	30.2

reputation and a voter's social network influence on overall voting behaviour. Finally, in Section 4 we discuss the results and draw conclusions.

2. Election data and analysis

2.1. Data

In this section, we analyse the election data of a democratic voting system. The focus of the data collected was on the election results from seven democratic countries, in particular on the status of each country's leader and party, whether newly elected, re-elected once, or re-elected multiple times. The countries chosen have had at least 20 democratic elections and have a near universal suffrage. Two of the oldest democratic voting systems are France and the USA allowing us to analyse a large number of election results, while the five original Commonwealth countries Australia, Canada, Republic of Ireland, New Zealand, and the UK all have similar government structures[1]; for the sources for the data see Refs. [8,9,39–55]. The election data for other democratic countries were not considered as they have had few general elections, yielding highly, volatile election averages. The election results are collected up until the election year 2011, as the leader's term in office had not come to an end.

As this research focuses on analysing the status of a leader and party, we simplify each country's government system to a two-party system, where the two parties are the dominant party and the opposition. For some countries like the USA, only two parties have been in power, therefore making their political system a good fit for our research. Other countries for example France, have had numerous political parties in charge throughout history and to fit our classification of a two-party system to the French voting history, we subjectively classified the parties to have either left or right wing political views. Doing so allows us to categorise them into a two party system. The party in charge for more than 50% of the time is defined as the dominant party.

In Tables 1 and 2, we list the percentages that the dominant party is elected and the chance of a party leader is re-elected for the original Commonwealth countries and France and the USA, respectively. Percentages are calculated in terms of number of elections.

In both tables we see that the dominant party was elected between 52% and 66% of the time for all countries. However, the biggest differences in the election data can be seen from the re-election percentages of the democratic leadership varying from around 47% to 66% for the five Commonwealth countries but only between 21% and 30% for France and the USA. Even though the leaders of the USA can only be re-elected once since the 1950s, prior this only President Roosevelt was re-elected multiple times.

It is perhaps not unexpected that the election percentages for the five original Commonwealth countries are almost identical given that they have similar election systems. However, that France and the USA have a dominant party in charge with a percentage similar to the five original Commonwealth countries, but have a significantly lower re-election percentages raises several interesting questions. In order to analyse and explore the differences and the effect of previous elections in the re-election percentages we will first try and fit a simple Markov model to the data.

2.2. Markov model

To investigate the data and identify further possible underlying mechanisms for the voting behaviour observed in the data in Section 2.1, we fit a simple three-state stochastic Markov model describing the time evolution from one election to the next keeping track of the probabilities of election/re-election and re-election more than once, of a democratic leadership and also of the dominant and opposition party. The Markov model allows us to analyse the voter behaviour in each country and what this means in terms of the underlying voter behaviour for each electorate. The goodness-of-fit of the model's results to the data are also investigated.

[1] Although South Africa was part of the original Commonwealth, we have decided to exclude South Africa due to the apartheid system during the 20th century.

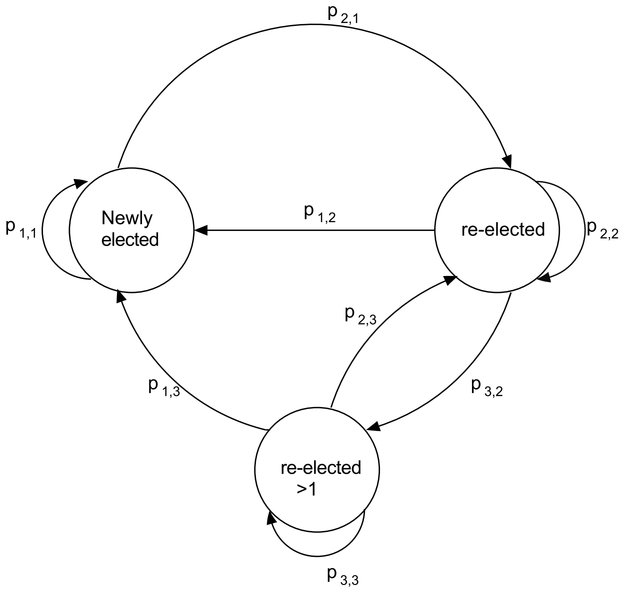

Fig. 1. An overview of the Markov model showing the three states of a leader/party and the corresponding transition probabilities.

The Markov model is set-up as follows; see Fig. 1 for a diagram of the model depicting the transition probabilities from each of the three states. Let $\mathbf{x}_n \in \mathbb{R}^3$ be the three probabilities of a democratic leadership or party being newly elected, re-elected once and re-elected multiple times at each election $n = 1, 2, 3, \ldots$, i.e.

$$\mathbf{x}_n = [\text{Probability newly elected, Probability re-elected once, Probability re-elected multiple times}]^T,$$

where T denotes the transpose of a vector. The probabilities $\mathbf{x}_n$ are updated at each election via the Markov model based on the previous election, i.e.

$$\mathbf{x}_n = \mathbf{P}\mathbf{x}_{n-1}, \tag{2.1}$$

where $\mathbf{P}$ is a probability matrix describing the transition probabilities between the three states, given by

$$\mathbf{P} = \begin{bmatrix} p_{1,1} & p_{1,2} & p_{1,3} \\ p_{2,1} & p_{2,2} & p_{2,3} \\ 0 & p_{3,2} & p_{3,3} \end{bmatrix}.$$

Let us briefly describe the choice of the probability matrix and the diagram of the model in Fig. 1 in terms of leaders being elected. At the start of each election history there will always be a newly-elected democratic leader. At the next election, with a probability $p_{1,1}$ another newly-elected leader is possible or the original leader is re-elected with a probability $p_{1,2}$, where $p_{2,1} = (1 - p_{1,1})$ since these are the only two possibilities. Once a leader has been re-elected once, there are three possible outcomes at the next election, namely a new leader is elected (with probability $p_{1,2}$), the leader is re-elected once with probability $p_{2,2}$, or the leader is re-elected more than once with probability $p_{3,2}$. As these are the only three possibilities for the outcome, $p_{3,2} = (1 - p_{1,2} - p_{2,2})$. The transition of $p_{2,2}$ describes when a leader has already been re-elected once, but cannot finish their term, then an unelected leader takes their place. If this unelected leader is elected in the following election, it is considered they are re-elected for the first time, not just elected, as they were already in power before the election. Once a democratic leader has been re-elected more than once, the following are possible: the leader is re-elected again with probability $p_{3,3}$, which is given by $(1-p_{1,3}-p_{2,3})$, a leader goes to being just re-elected once with probability $p_{2,3}$ (which occurs when an unelected leader comes to power during a term), or a newly elected leader is elected with probability $p_{1,3}$. This model is also suitable to analyse the election/re-election of the dominant and opposition party.

The transition probabilities $p_{2,2}$ and $p_{2,3}$ are included, as we are interested in uncovering fundamental mechanisms for elector behaviour and one possible influence is that being in power is significantly more advantageous. Hence, if an unelected leader comes to power before the next election, then the benefit/hinderance of being in power during the election should be taken account of.

In order to fit and calculate the probability transition matrix $\mathbf{P}$, for each country we calculate the entries of $\mathbf{P}$ via the equation

$$p_{ij} = \frac{m_{ij}}{m_i}, \tag{2.2}$$

where i and j denote one of the three states, m_{ij} is the total number of leaders up to a certain election n who made the transition from state i to state j from the data, and m_i is the total number of leaders in state i also from the data [56]. This

Table 3
Listed are the re-election probabilities of a leader for the original Commonwealth countries calculated from the steady state equilibrium distributions **x**, the re-election rates collected from the data and the *p* value which describe the goodness-of-fit of the three-state Markov model to the data.

	Australia	Canada	Rep. of Ireland	New Zealand	UK
Prime Minister's re-election probability given by the probability vector **x** (%)	52	65	57	68	50
Prime Minister's re-election percentages from the election statistics (%)	51	63	53	66	48
p	0.99	0.97	0.88	0.97	0.93

Table 4
Listed are the re-election probabilities of a leader for France and the USA calculated from the steady state equilibrium distribution **x**, the re-election rates collected from the data and the *p* values which describe the goodness-of-fit of the three-state Markov model to the data.

	France	USA
Prime Minister's re-election probability from the probability vector **x** (%)	21	30
Prime Minister's re-election percentages from the election statistics (%)	21	30
p	0.92	0.99

Table 5
Listed are the re-election probabilities of the dominant party for the original Commonwealth countries calculated from the steady state equilibrium distributions **x**, the re-election rates collected from the data and the *p* value which highlight the goodness-of-fit.

	Australia	Canada	Rep. of Ireland	New Zealand	UK
Party's re-election statistic from the probability vector **x**	70	67	61	76	56
Party's re-election statistic given in percentages (%)	68	66	57	74	55
p	0.97	0.97	0.87	0.92	0.95

leads to a maximum likelihood fit of the Markov model to the data and is equivalent to carrying out a standard Least Squares fit of the model with the data [56].

As the probability matrix **P** is column stochastic and irreducible, a standard fixed point theorem guarantees the existence of a unique, globally attracting fixed point, i.e. the steady state equilibrium distribution of the Markov model as the number of elections tend to infinity ($n \to \infty$) satisfies the steady state relation

$$\mathbf{x} = \mathbf{P}\mathbf{x}, \tag{2.3}$$

where **x** is the steady state probability vector. Solving (2.3) is equivalent to finding the unit eigenvector of **P** and re-normalising the eigenvector so the probabilities add up to one. We then test the goodness-of-fit by carrying out a standard Chi-square test.

2.3. Markov model results

We present the analysis of the fitted Markov model to each country's election data of a leader and party. Listed in Tables 3 and 4 are the steady state equilibrium distributions **x** calculated from the three-state Markov chain, the re-election percentages from the data from Section 2.1 and the goodness-of-fit *p* evaluated from the Chi-square test. A comparison of the leader and party re-election statistics shows that there is little variance in the results.

In order to test the goodness-of-fit of the Markov model and the choice of a three-state model, a Chi-square test was performed on all of the calculated steady state equilibrium distribution results for a two- up to eight-state Markov model, see Fig. 2 for an overview of the results. The maximum number of states considered is eight, as no leader was elected/re-elected more than eight times. The two-state model (where the states are newly elected and re-elected) is a very good fit for re-election data for Australia, France and the USA. However, the two-state model is a poor fit for New Zealand, Republic of Ireland and the UK with the goodness-of-fit, as given by the *p*-values, lies between 0.1 and 0.7. On the other hand, the goodness-of-fit for a three-state Markov model increases to $0.88 < p < 1$ for all countries. As expected, the goodness-of-fit improves as the number of states increased, but this is only minimal. Hence, we believe the three-state Markov model captures the key features of all the countries' voting behaviours, whilst also remaining simple enough to yield some fundamental mechanisms and similarities of various country's voting behaviours.

Finally, we focus on the election/re-election of the political parties. Listed in Tables 5 and 6 are the steady state equilibrium distributions **x** for a party calculated by the three-state Markov chain, the re-election percentages from the data from Section 2.1 and the goodness-of-fit *p* results. Again, a comparison of the two party election statistics shows that there is not much variance in the results.

The goodness-of-fit is again calculated for all countries for a two- to eight-state Markov model, see Fig. 3 for an overview of the results. It can again be seen that the two-state Markov model does not fit the data well; for example the goodness-of-fit for the Republic of Ireland is evaluated at $p \sim 0.5$. However, for France and the USA the two-state model is again a good fit for the data, with the goodness-of-fit ranging between $0.8 < p < 0.9$. The results for a three-state Markov model increases

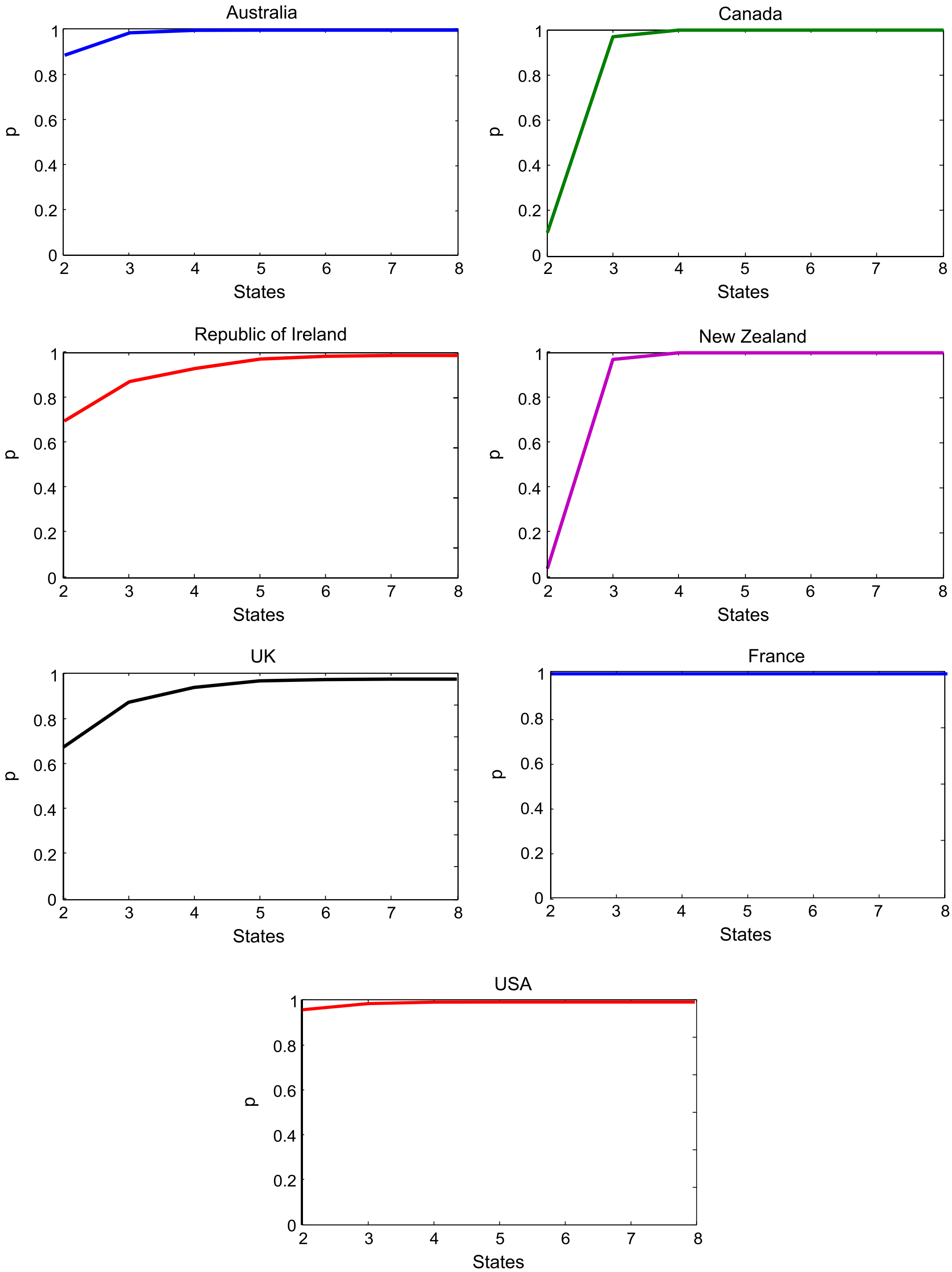

Fig. 2. An overview of the goodness-of-fit given by the p value and the number of states used in the Markov model of the results for the re-election of leaders.

to the values between $0.87 < p < 0.98$. There is only a slight increase for the results of higher states. Hence, in order to have a single Markov model that sufficiently captures the key features of the data in all of the countries a three-state model is adequate.

Overall, we have found that a simple three-state Markov model dependent only on the previous election, provides a good fit of the election data and suggests that the "memory" in the system beyond the current state does not matter. Given the similarities of the French and the USA voting behaviour compared to that of the Commonwealth countries it appears

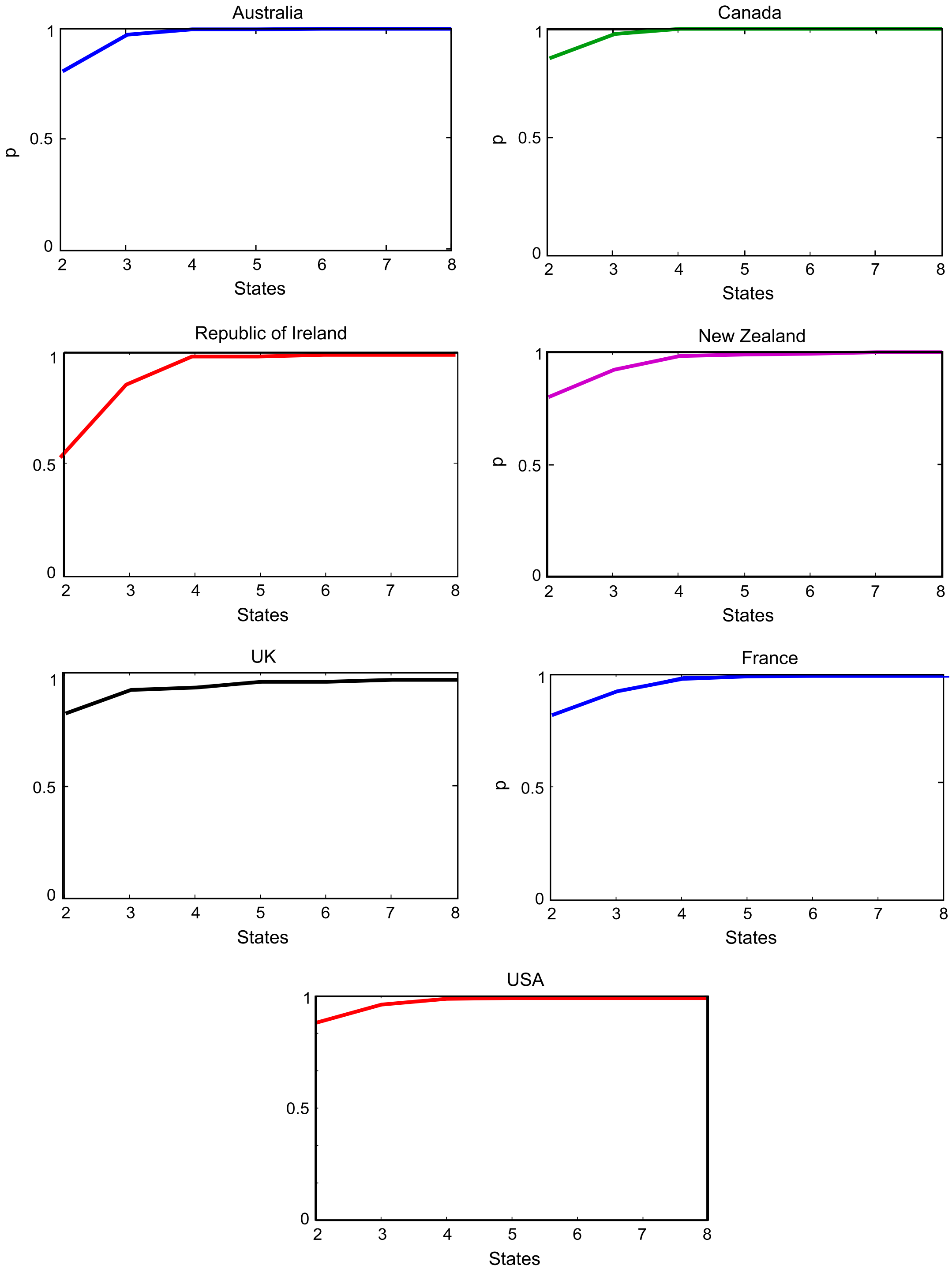

Fig. 3. The steady state equilibrium distribution for voter behaviour for political parties in the Commonwealth countries, France and the USA in the three-state Markov model.

that being in power is a major benefit to leaders (which is not influenced by the leader's party) in the Commonwealth countries and is a hinderance for leaders in France and the USA. This analysis raises several interesting questions. What are the fundamental influences on voting behaviour? Since we have ignored differences in the constitution of each country, we look at possible mechanisms that would be generic. As mentioned by other researchers [35,37], one hypothesis for the differences in the voter behaviour is that the country's voters are more strongly influenced by their family/friends/social

Table 6
Listed are re-election probabilities of the dominant party for France and the USA calculated from the steady state equilibrium distributions **x**, the re-election rates collected from the data and the p value which highlight the goodness-of-fit.

	France	USA
Party's re-election statistic from the probability vector π	69	61
Party's re-election statistic given in percentages (%)	67	60
p	0.93	0.97

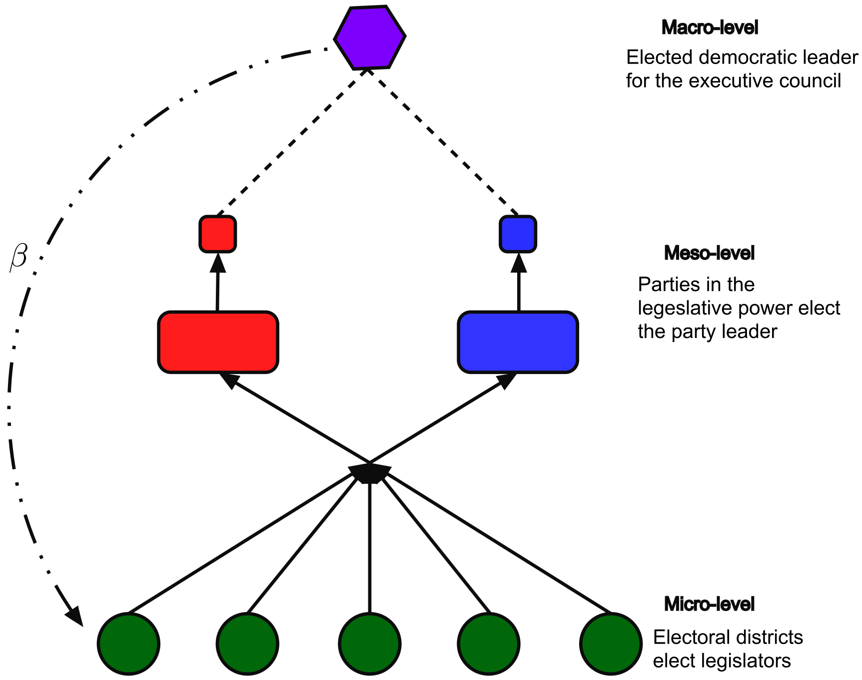

Fig. 4. A schematic diagram of the different levels on the voter model. In the micro-level, there are an odd number of electoral districts where individuals vote for an legislator. The elected legislators are grouped into the meso-level, creating two political parties, where they vote internally for a party leader. At the macro-level the democratic leader is selected on the basis of the size of the political parties in the meso-level. The success score of the democratic leader's term in office, β, is fed back into the micro-level.

network. Another possible explanation for the different re-election rates of leaders is the impact of their reputation on voters. As shown in research [25] voters perceive the leader to be either successful or unsuccessful, which could have various degrees of impact on the voters. In order to construct a model to examine these two hypotheses, we will need to construct a voting model that incorporates the fundamental voter behaviour mechanism described by the three-state Markov model and the influence of family/friends/etc. and reputation of a political leader on individual voters.

3. Voting model

In the previous section, the voting behaviour of each county was analysed, showing the Commonwealth countries to have similar voting behaviour, as do France and the USA. A possible explanation of these similarities and differences of voter behaviour is due to the influence of family/friends/etc. and leader's reputation on individual voters. Hence, in order to explain the election data we need a model that incorporates these influences on individual voters but also has the underlying mechanism that voters decide who to elect based on who was previously in power. Since these mechanisms operate at different stages in an election one needs to construct a descriptive model of the election process.

The voting model is broken down into three stages or levels, namely the micro-, meso- and macro-levels; see Fig. 4 for an overview of the model. At the micro-level, individual voters are grouped into N_c equal-sized electoral districts with N_v voters in each district. The voters in each electoral district are modelled using a 2-D modified version of Galam's model that incorporates influences from nearest social neighbours, their previous vote and the success score β from the previously elected leader. In each of these electoral districts, each voter elects one of two parties (denoted by -1 and $+1$) and the mean of the vote in each district determines which legislator is elected into the meso-level, where two political parties are formed.

The political party in charge before the election keeps the democratic leader as the party leader only if their term was successful; the success of a leader's term in office is randomly scored to be either successful or unsuccessful. If it was unsuccessful, the party votes for a new party leader. Each party elects a leader based on a randomly assigned leadership score from a standard Uniform distribution assigned to each legislator (this is only re-assigned if a new legislator is elected, i.e. there has been a change of party elected in an electoral district), and on each legislator's political complexion, which is based on the mean vote in their electoral district. A leadership battle is then carried out, where the candidates are the two legislators with the highest leadership score. As most of the legislators elected are very secure in their seats, they are likely to have strong views in order to reflect their electorate and hence will look for a party leader who also reflects those strong views and vice-versa. Therefore, all the other legislators vote for the candidate who has the closest political complexion to their own.

The leader of the party with the most legislators is selected as leader for the executive council in the macro-level. Their reputation, which is either successful or unsuccessful denoted as β_{suc} and β_{unsuc}, respectively, is determined based on whether a random number is lower/higher than the leader's leadership score, and influences the voters in the following election. Only two types of reputation are considered, as research has shown that voters perceive the leader's term in office to be either successful or unsuccessful, see Ref. [25]. We note that the implementation of a negative reputation on individual voters and that a voter's next vote is partially based on their previous vote, reflects the fundamental mechanism uncovered by the Markov model since if a leader is re-elected and has a negative impact on voters then in the following election their support will decrease.

In order to calculate the re-election averages for leaders and parties, we simulate 50 sequential elections to yield one re-election average for a leader and another re-election average for the parties. We then repeat the simulation 20 times to yield 20 re-election averages for leaders and 20 re-election averages for the parties. We then average the re-election averages over these 20 different simulations. Since we are averaging over averages, we may compute the standard error of the mean to provide confidence on the statistical significance of our results and compare them with the election statistics in Section 2.1.

We will now describe in more detail the modification of the 2-D Galam model [57–59] that we use in each electoral district at the micro-level of the model. A tension T_i is created between an individual voter's party preference and their neighbour's voting decision. If the individual voter and their social neighbours are likeminded then there is no tension between them, whereas a difference in party preference creates a larger tension between them. The tension for each voter is given by

$$T_i = -\frac{J}{2} s_i \sum_{\langle il \rangle} s_l - \mu s_i, \tag{3.1}$$

where $s_i = \pm 1$ is the vote of each voter i, J is the strength of the nearest social neighbours's influence, $\langle il \rangle$ restricts the individual voter i to be influenced by its l nearest neighbours, where $l = 2d$, and μ is the strength of the external influence from the leader. A small influence of a voter's nearest neighbours corresponds to a small value of J. The external influence is chosen to reflect how the electoral district previously voted and the current leader's political party and their reputation. In particular, if an electoral district voted the same way as the overall outcome of the election, then it decreases the tension in the system. On the other hand, if an electoral district voted counter to the overall outcome of the election then there is an increased tension for the voters to choose the opposition. Hence, the external influence is defined as

$$\mu = \text{sign}(\bar{s}^{\text{prev}}) - \beta \ \text{sign}\left(\sum_{j=1}^{N_c} \bar{s}_j^{\text{prev}} \right), \tag{3.2}$$

where N_c is the total number of electoral districts, $\bar{s}^{\text{prev}}$ the mean vote in the previous election of the voter's electoral district, $\bar{s}_j^{\text{prev}}$ is the mean vote in the previous election of *all* electoral districts, and β is a number given by

$$\beta = \begin{cases} \beta_{\text{suc}} & \text{leader's term was successful} \\ \beta_{\text{unsuc}} & \text{leader's term was unsuccessful.} \end{cases}$$

Both β_{suc} and β_{unsuc} have a negative effect on voters if chosen to be positive values. In order to determine if a leader was successful or not, we draw a random number from the standard Uniform distribution and determine that the leader was successful if this random number is less than the leadership score of the leader and unsuccessful otherwise.

We use the Boltzmann distribution to determine the probability that an electoral district is in a certain configuration, i.e. voted in a certain manner

$$P(s) = \frac{e^{-\sum_{i=1}^{N_v} T_i}}{\sum_{s=\pm 1} e^{-T_i}}, \tag{3.3}$$

where N_v is the number of voters in an electoral district. We have set the "temperature" and "Boltzmann constant" in Boltzmann distribution to unity. The Boltzmann distribution is chosen, as the most likely voter configuration is found by maximising the probability (3.3) that is equivalent to minimising the total tension ($\sum_{i=1}^{N_v} T_i$) for each electoral district. This minimisation is carried out using the Metropolis–Hastings algorithm.

For the first two elections, the external influence μ is set to zero so that the voters are only influenced by their neighbours, as in the first election there is no leader. For the third and later elections, the external influence μ has an effect on the voters.

3.1. Initial analysis and calibration

Before we run simulations of the voting model, we first need to narrow down the parameter space we wish to explore, as it is not clear what values one should take for the nearest social neighbour influence parameter J or the values the leaders influence β_{suc} and β_{unsuc}. For this research a double periodic 2-D lattice is chosen, where the voters are influenced by their four closest neighbours to the north, east, south and west denoted as $z = 4$.

Let us first describe the two general types of behaviour we expect of the model. For a small coupling constant J, we expect the tension of each voter to be governed mostly by the leader's influence. Starting from a Uniform random distribution of voters, the mean vote $\bar{s}$ in an electoral district will be approximately zero. Through (3.2) the external influence will be oscillating from positive to negative values due to random fluctuations but we expect the mean of the external influence to be approximately zero. Based on this the mean vote in an electoral district will remain close to zero. For a large coupling constant J, the influence of the leader can be neglected leading to clusters of similar minded voters. In this case, we expect the dominant party to be in charge all the time. Before we carry out simulations of the voter model we will first analytically look at trying to understand how the influence of the leader will change the mean voting behaviour for a single and multiple electoral districts.

We consider a single electoral district and find the approximate tension for each voter, T_i as

$$T(s_i) \approx -\left(\frac{Jz}{2}\bar{s} + \mu\right) s_i =: T_{\text{approx}} s_i, \tag{3.4}$$

where the nearest neighbour interaction is approximated by the mean vote and μ is given by (3.2). Since the tension of each voter is decoupled, the probability for a single voter s_i is given by

$$p(s_i) = \frac{e^{-T_{\text{approx}} s_i}}{e^{-T_{\text{approx}}} + e^{T_{\text{approx}}}}, \tag{3.5}$$

and in order to have consistency with the mean-field approximation, the expected mean value vote calculated from the probability distribution by (3.5) should be equal to the mean vote, i.e.

$$\bar{s} = \sum_{s_i=\pm 1} p(s_i) s_i = \tanh(T_{\text{approx}}). \tag{3.6}$$

Hence, we have the mean field equation

$$\bar{s} = \tanh\left(\frac{Jz}{2}\bar{s} + \mu\right), \tag{3.7}$$

to solve. The effect of the external influence μ, will lead to an $\bar{s}$ that is the same sign as μ; see Fig. 5 for a graphical sketch of the solutions of (3.7) for $\mu > 0$ and $\mu < 0$ and Jz small/large. Hence, for small β, there is an increased preference to vote for the same overall outcome (± 1) as before and a constantly elected dominant party is expected to occur. We also observe that as the coupling constant increases the non-trivial mean solutions of (3.7) tend to ± 1.

One may also consider the case where all the electoral district votes with the same mean $\bar{s}$. In this case, we are able to consider the effect of multiple runs of the model. We proceed by approximating μ by the smooth function

$$\mu = \text{sign}(\bar{s})(1-\beta) \approx \tanh(\alpha\bar{s})(1-\beta), \quad \alpha \in \mathbb{R}^+, \tag{3.8}$$

where $\alpha \gg 1$. A necessary condition for a non-trivial solution of (3.7) is that the derivative of the right hand side of (3.7) at the origin $\bar{s} = 0$ is greater than unity. This leads to the condition

$$\frac{Jz}{2} + \alpha(1-\beta) > 1. \tag{3.9}$$

Provided $0 < \beta < 1$ and α sufficiently large the model will always attain this condition irrespective of the value of the coupling constant J. Correspondingly, if $\beta > 1$ then for α sufficiently large we never expect to attain this condition. Hence, we do not expect to see a critical value of the coupling constant in the numerical simulations but the coupling constant may still have an effect on the re-election rates.

Although there does not appear to be a critical parameter there is a clear affect of varying the value of the coupling constant and reputation parameters β on the mean field approximation. In particular, a positive reputation parameter $\beta > 0$ has a negative impact of voters, thus reducing the chance of a leader to be re-elected, whereas for $\beta < 0$ increases their chance. We also observe that as the coupling constant increases the non-trivial mean solutions of (3.7) tend to ± 1. Therefore, for the model to recreate the different voting behaviours observed in Section 2.1 both parameters need to be varied.

3.2. Political voting model results

For all our simulations, we choose the number of electoral districts $N_c = 11$ with the number of voters $N_v = 100 \times 100$. The voters in each electoral district are set on a torus, where each voter is influenced by the four closest neighbours to the

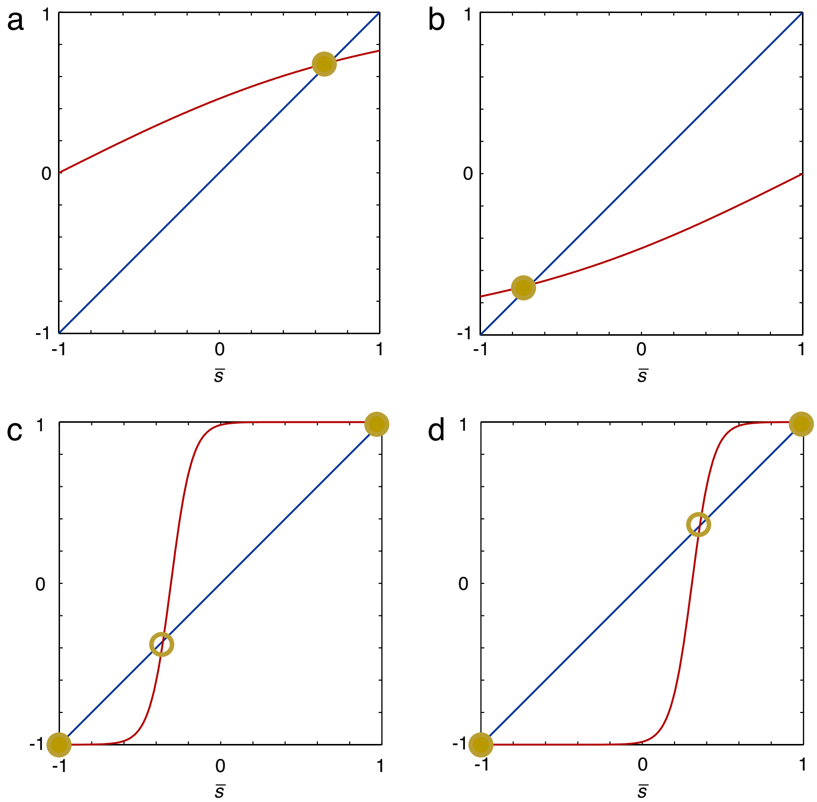

Fig. 5. Blue line is the left hand side (3.7) and red line is the right hand side of (3.7). A solution of (3.7) corresponds to intersections of the two graphs denoted by circles for (a) $Jz < 1$ and $\mu > 0$ (b) $Jz < 1$ and $\mu < 0$ (c) $Jz > 1$, $\mu > 0$ and (d) $Jz > 1$, $\mu < 0$. Linear stability is denoted by full circles and linear instability is denoted by empty circles. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

north, east, south and west. As shown in Section 2.3 when analysing voting behaviour a distinction between leader and political party election needs to be made.

In order to calculate the re-election averages for leaders and parties, we simulate 50 sequential elections to yield one re-election average for a leader and another re-election average for the parties. We then repeat the simulation 20 times to yield 20 re-election averages for leaders and 20 re-election averages for the parties. The standard deviation (taken over the 20 simulations) for all the results lies between 0 and 0.2451 for the re-election of a leader and between 0 and 0.1949 for how often a party is elected. Computing the standard error of the mean yields at worse a standard error of approximately 3.5%. Hence, when analysing the results we will generally be looking for differences in the re-election averages of greater than 7%. However, it is rare to observe such high deviations, and we find the mean of the standard deviations for the re-election rate of a leader is 0.0329 and for a party election 0.0213 yielding a standard error of approximately 0.5%.

We start by first looking at the effect of the leadership influence, β_{suc} and β_{unsuc}, and the nearest neighbour influence, J, on the election averages from the model. In Fig. 6 a two parameter plot of the re-election rate of a leader varying the parameters J and β_{unsuc} is shown, where the successful reputation parameter is set to $\beta_{suc} = 0$, i.e. a successful leader has no impact on the voters. In the figure, each square represents the value of the axis label and not the value in between. We see that when the unsuccessful reputation has no impact, i.e. $\beta_{unsuc} = 0$ around 60% of leaders are re-elected consistently. In particular, the results show that a successful and unsuccessful leader can be re-elected multiple times. For $\beta_{unsuc} = 0.1$ there is a drop to around 50% in the leader's re-election rate and there is a higher chance for a random voting pattern. An analysis of the election histories show that for the case when $\beta_{unsuc} = 0.1$ an unsuccessful leader was able to be re-elected once, but this was a rare occasion. For a stronger unsuccessful reputation $\beta_{unsuc} > 0.1$ there is an increase in leader's being re-elected. This is due to voters not re-electing an unsuccessful leader, whereas a successful leader is re-elected multiple times. Therefore, there were more successful leaders. As these results still do not produce the re-election rates similar to the French or American countries, we require that a successful reputation must have a slight negative impact on voters.

In Fig. 7, we set $\beta_{unsuc} = 0$, i.e. the unsuccessful reputation is set to have no impact on voters while we vary the coupling constant J and the successful reputation β_{suc} and plot the re-election rate of the leaders. Again, each square represents the value of the axis label and not the value in between. Note the successful reputation has a negative impact on voters. In this case we see that there is a very low re-election rate for almost all values. In particular, only if a successful reputation is set

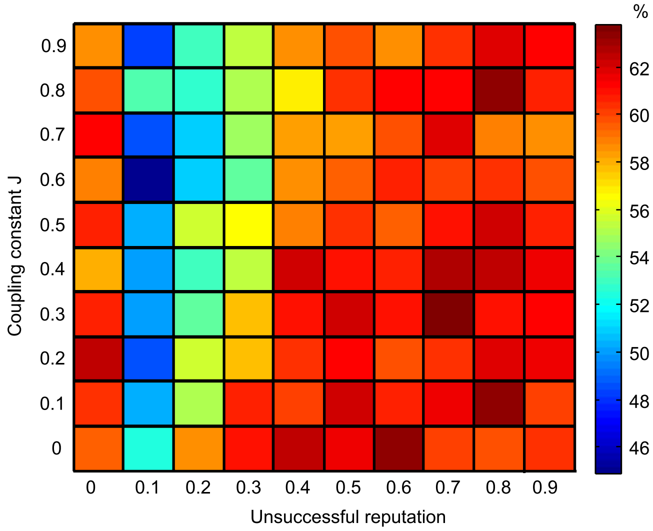

Fig. 6. A plot of the average re-election rates of the voter model for leaders, where the successful reputation is set at $\beta_{suc} = 0$. The colours indicate the re-election rate of a leader.

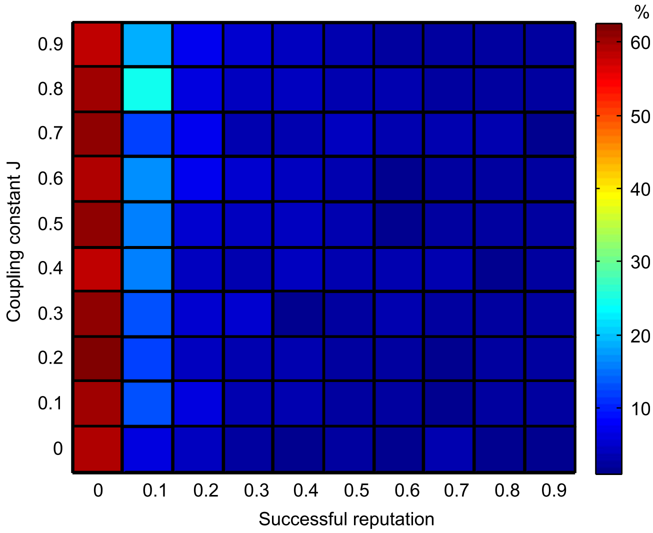

Fig. 7. A plot of the average re-election rates of the voter model for leaders, where the unsuccessful reputation is set at $\beta_{unsuc} = 0$, where the x-axis is the negative impact of a successful leader on voters.

to have no impact are there any re-elected leaders, which in turn means that a successful reputation cannot have a stronger negative impact on voters than an unsuccessful reputation.

In Fig. 8 the results on the left have the coupling constant J set to zero, i.e. voters are not influenced by their friends'/family's party preference and vary the parameters β_{suc}, β_{unsuc} whereas in the results on the right in Fig. 8 the same parameters are varied but the coupling constant is set to one, i.e. voters are likely to adopt their neighbours' party preference. Again, each square represents the value of the axis label and not the value in between. In the case $J = 0$, the mean field theory predicts that the mean vote should be equal to $\tanh(\pm(1-\beta))$ and hence the mean will converge to zero as β tends to one. Interestingly, in both cases for $J = 0$ and $J = 1$, we observe a clearly well defined sharp transition from a 0% re-election rate to a 60% re-election rate. We do not have a good explanation of the reason this transition occurring from the mean field theory.

Comparing these two figures, we see that the greater the influence of friends'/family's party preference, the less likely it is for a leader to be re-elected for large values of β_{unsuc} and β_{suc} equal to either 0.2 or 0.3. This is a surprising result since one might expect lock-in of votes to occur and an increase in the leaders being re-elected. However, due to the effect of a leader's reputation, lock-in does not occur.

Next we carry out a trial-and-error calibration of the model to the election data of the Commonwealth countries and then vary one of the parameters J, β_{suc}, β_{unsuc} while keeping the others fixed to see if the model can reproduce similar election averages to France and the USA.

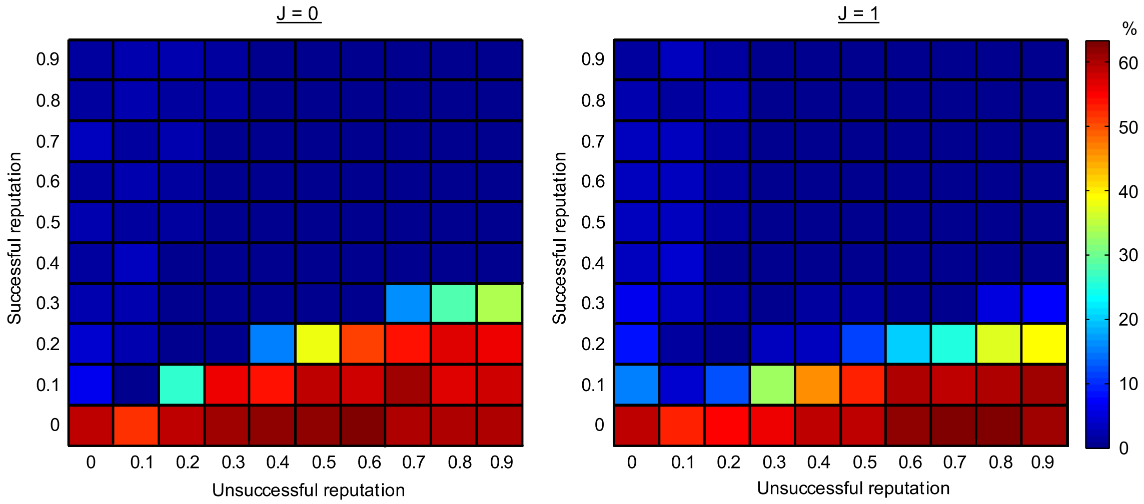

Fig. 8. A plot of the average re-election rates of the voter model for leaders, where in the figure on the left, the coupling constant is set at $J = 0$, so that voters are *not* influenced by their neighbours. In the figure on the right, the coupling constant is set at $J = 1$, such that the voters are likely to take on their neighbour's party preference.

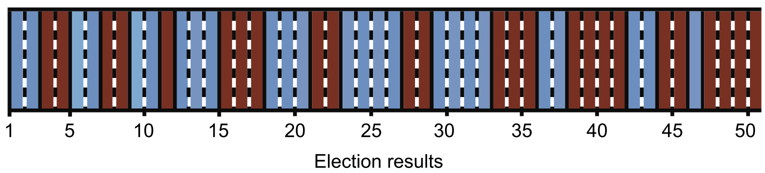

Fig. 9. Overview of one electoral history for Commonwealth countries. The light blue and dark red fields indicate the leader's political party. A solid line dividing the elections indicates a new leader has been elected, whereas a dotted line indicates a re-elected leader. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

We fix the coupling constant set for both model fits at $J = 0.48$ (just above the critical coupling constant for a system with no external influence), such that voters are slightly influenced by their friends'/family's party preference. We match the model's results for the re-election of a leader and how often a party was in power to the data in Section 2.1 for the Commonwealth countries. Setting $\beta_{suc} = 0.03$ and $\beta_{unsuc} = 0.4$, we find that the dominant party is in power for approximately 54% of the elections and approximately 56% of leaders being re-elected; see Fig. 9 for one realisation of an election history. This matches the election averages for all the Commonwealth countries where one party is found to be in charge for approximately 57% of the elections with around 59% of leaders being re-elected once. We see in Fig. 9, that for these parameters it is possible and fairly common for a leader to be re-elected twice but we do not observe more re-elections.

The re-election rates of the leaders for France and the USA are half those of the Commonwealth countries. For the model to yield re-election rates similar to France and the USA, one possibility is that we fix $J = 0.48$ and change the reputation scores to $\beta_{suc} = 0.08$ and $\beta_{unsuc} = 0.3$. On average, approximately 26% of French and American leaders were re-elected, and the dominant party was on average 59% of the time in charge. In Fig. 10, we see one election history of the model and that on average one party is in power for approximately 62% of the time and that 27% of leaders are re-elected. We do see a leader being re-elected twice, but this is rare and that most of the time a new leader and party is elected. The biggest change in the parameters from the Commonwealth fit, is in the value of the negative reputation score β_{suc} where we have increased the value to 63% from the Commonwealth value suggesting that a possible explanation for the differences in the countries is that successful leaders in countries similar to France and the USA have a bigger negative influence than in the Commonwealth countries. Hence the re-election rates of the leaders in France and the USA are significantly lower than for the Commonwealth countries.

We next fix, β_{suc} and β_{unsuc} to the values obtained for the Commonwealth countries and vary J to carry out a detailed statistical analysis showing the effect of the coupling constant on the re-election averages. For this analysis, we increase the number of simulations of the model from 50 to 100 in order to decrease the standard error of the mean. In Fig. 11, the thin blue line describes the trend of the averages of how often the dominant party was in charge, where we see an increase for the mean re-election rate by 1.96% as the coupling constant J increased from 0.1 to 2. The blue non-filled circles around the trend line of the average party election rate are the results of the mean party election obtained from the model, including the standard error bars, and the green dashed lines are the confidence bounds, indicating that with a 96% confidence a single re-election mean from one simulation of 100 sequential elections will lie in that corridor. The thick black line shows the trend of the average of the re-election rates of leaders, where the black filled dots are the results obtained from the

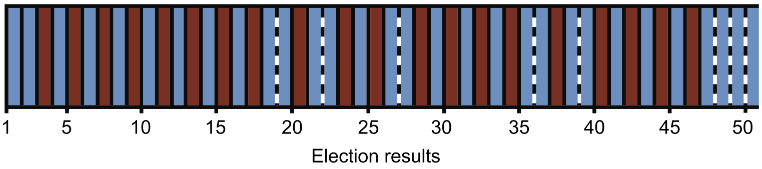

Fig. 10. Overview of one electoral history for France and the USA. The light blue and dark red fields indicate the leader's political party. A solid line dividing the elections indicates a new leader has been elected, whereas a dotted line indicates a re-elected leader. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

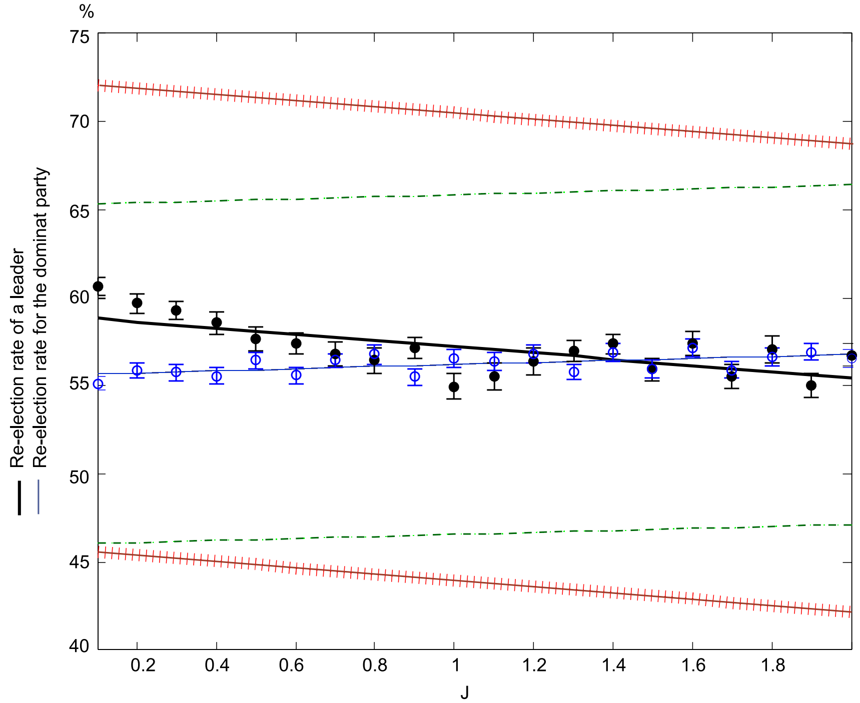

Fig. 11. A plot of the average re-election rates of the voter model for leaders and the dominant party, where the success scores for the Commonwealth countries are kept the same, while the coupling constant J increases steadily. The thin blue line shows the trend of how often the dominant party is in charge, where the non-filled dots are the mean results from the model including the standard error bars and the green dashed lines are the confidence bounds. The increase of the dominant party election is 1.96% over $J \in [0.1, 2]$. The thick black line shows the trend of how often a leader was re-elected, with the filled dots describing the results obtained from the model and the red crosses highlight the confidence bounds. The average of how often a leader is re-elected decreases by 5.74% over $J \in [0, 2]$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

model including the standard error bars and the red crosses describe the confidence bounds, again indicating that with a 96% confidence a mean from 100 sequential elections will lie in that corridor. It can be seen that increasing the influence of the coupling constant J on voters decreases the mean of the re-election averages for the leaders and we observe a drop in the confidence intervals (shown in red in Fig. 11). In order to test the statistical significance that the linear relationship exists we carry out a t-test and find the p-values for the leader and party re-election rate are less than 0.05 (0.00058 and 0.0062, respectively). Hence, we may reject the null hypothesis that the slope fitted is zero for both re-election averages.

Finally, we fix the β_{suc} and β_{unsuc} for the values obtained for France and the USA and vary J, which as in the previous case, the re-election rate of a leader decreases with the increase of the coupling constant J, while the party re-election increases. Again, we increase the number of simulations of the model from 50 to 100 in order to decrease the standard error of the means. In this case the decrease of the leader re-election rate is around 14.37%, while the party re-election rate increases by 2.46%, as shown in Fig. 12. We carry out a t-test for the null hypothesis that the fitted slope is zero, and find that the p-values for the leader and party re-election rate are less than 0.05 (0.0341 and 0.0349, respectively) and again we can reject the null hypothesis.

These results show that a stronger impact of social neighbours J on voters has consequently a negative influence on the leader's re-election rate (albeit a weak influence), while having a slight positive influence of party re-election rate. This implies that, for a larger coupling constant J, there is a greater change of a political party changeover, than for a smaller coupling constant. Furthermore, this model suggests that the reputation of a leader and the impact of social neighbours is correlated. For example, for a stronger influence of the successful reputation as given in the France and the USA case

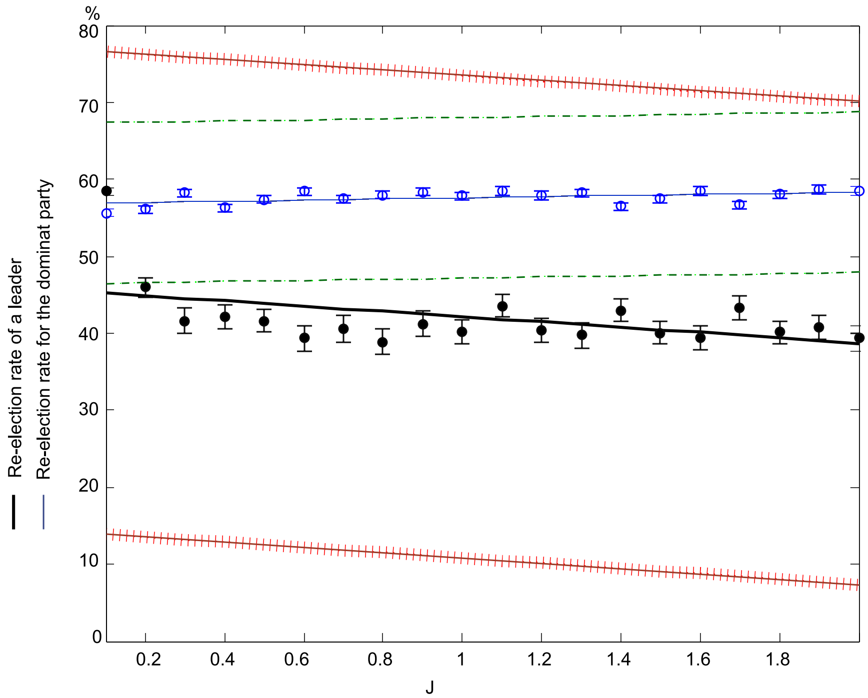

Fig. 12. A plot of the average re-election rates of the voter model for leaders and the dominant party, where the success scores for France and the USA countries are kept the same, while the coupling constant J increases steadily. The thin blue line shows the trend of how often the dominant party is in charge, where the non-filled dots are the mean results from the model including the standard error bars and the green dashed lines are the confidence bounds. The increase of the dominant party election is 2.46% over $J \in [0.1, 2]$. The thick black line shows the trend of how often a leader was re-elected, with the filled dots describing the results obtained from the model and the red crosses highlight the confidence bounds. The average of how often a leader is re-elected decreases by 14.37% over $J \in [0, 2]$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

enhances the impact of the influence of social neighbours. The model does support the theory that just the two reputation scores for a successful and unsuccessful leader are enough to understand the data. In all cases for the model to reproduce the election averages of the data, we require that an unsuccessful reputation has a far greater negative impact on voters than a successful (negative) impact on voters.

4. Conclusion

In this paper we have looked at the election averages for the leaders and political parties of seven different countries. It is clear that looking at just the re-election averages of parties and leaders while ignoring vastly different constitutions/cultures etc., one sees that the Commonwealth countries are all roughly similar but significantly different from France and the USA. In particular, it is found that the original Commonwealth countries re-elect around 58% of their leaders while France and the USA re-elect only around half as many.

Rather than just simply treating the election data as static, we fitted a time dependent three-state Markov model to investigate the effect of previous elections. It is found that the Markov model fits the data well suggesting that on average voters only consider the most recent past election in making their next vote. In order to further investigate the underlying voting mechanism, a descriptive dynamical model is created to test two possible theories namely, the influence of social nearest neighbours and a leader's reputation on voters. From this model, we find that while an increase in the influence of social nearest neighbours on individual voters has a negative impact on the re-election rates of leaders and an increasing impact on party re-election rate, the most important factor for the model to reproduce the election averages from the data is the two reputation scores for a successful and unsuccessful leader. Both these reputation scores have a negative impact on voters but crucially we require that the impact of a successful leader needs to be significantly less than an unsuccessful leader. Finally, it is believed that the impact of social neighbours and leader's re-election on individual voters are correlated.

From the literature, we find several authors support the hypothesis that voters in the USA are more influenced by family party preference, whereas other countries such as the UK are more influenced by parties that present clear political alternatives [16,35]. Furthermore, the possibility of an unsuccessful leader having a greater negative influence on voters than a successful one is also supported by several authors [27,29,31]. However, the major novelty of this work is to show that even if one disregards constitutional and cultural differences, the election averages can be explained by some simple rules implemented in a conceptual mathematical model. This does not mean that the precise context of a specific election is irrelevant rather that if we make the assumptions that *all* voters follow a simple set of general rules then we can reproduce the election data averages.

As shown by Chatterjee et al. [24], a comparison of countries with open list proportional election voting system behave in a similar way. The voting model in Section 3 could be adapted for such a system, where the hierarchy within the model would have to change. In the micro-level the parties put forward a number of candidates to be elected as the leader. Voters in the electoral districts would list the candidates in order of their preference and based on the overall vote a leader is elected. A comparison of the two different models could provide further information on impacts on voter behaviour and the influence of the type of election process used.

We highlight that this work tries to draw out a few fundamental mechanisms for voter behaviour from specific elections and that there may be other types of general behaviour of voters not investigated here. However, this work has shown that simple conceptual models are able to reproduce the gross averages from election data and support several hypotheses for the underlying rules governing electoral systems. We believe that the type of modelling carried out in this paper will help support and investigate several hypotheses that various researchers have found from analysing particular elections.

Acknowledgements

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